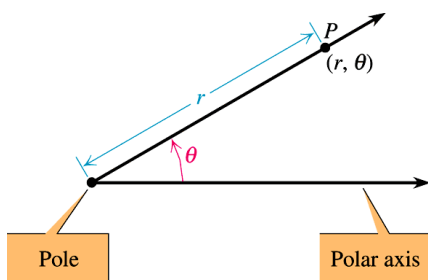


Polar Equations



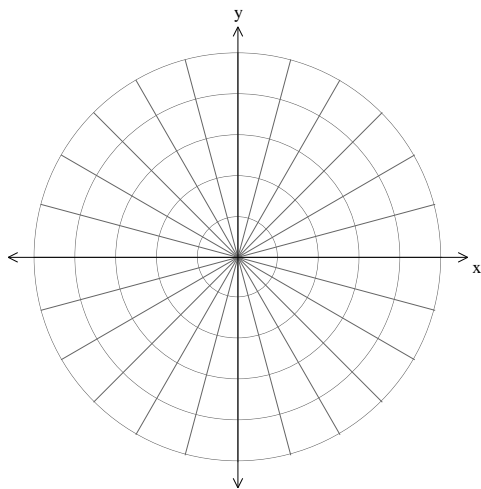
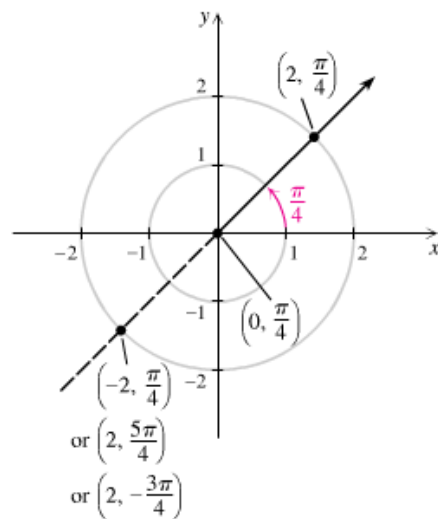
The polar coordinate system is based on a fixed point called the **pole** and a fixed axis called the **polar axis**. Points are represented by ordered pairs in the form (r, θ) , where r is the **directed distance** from the pole and θ is an angle whose initial side is the polar axis and whose terminal side contains the point. Typically, we choose the origin as the pole and the positive x -axis as the polar axis.

*To graph $(-r, \theta)$, you move in the opposite direction you would move to graph (r, θ) .

Polar coordinates are not unique. The points $(-2, \frac{\pi}{4})$, $(2, \frac{5\pi}{4})$, and $(2, -\frac{3\pi}{4})$ all name the same point.

Examples: Plot the points whose polar coordinates are given.

$A(3, \frac{\pi}{3})$, $B(-1, \frac{\pi}{6})$, $C(2, -\frac{7\pi}{4})$, $D(-5, -\frac{3\pi}{4})$, $E(4, \frac{\pi}{2})$, $F(-3, \frac{2\pi}{3})$



Polar-Rectangular Conversion Rules

- To convert (r, θ) to rectangular coordinates (x, y) , use $x = r \cos \theta$ and $y = r \sin \theta$.
- To convert (x, y) to polar coordinates (r, θ) , use $r = \sqrt{x^2 + y^2}$ and any angle θ in standard position whose terminal side contains (x, y) .

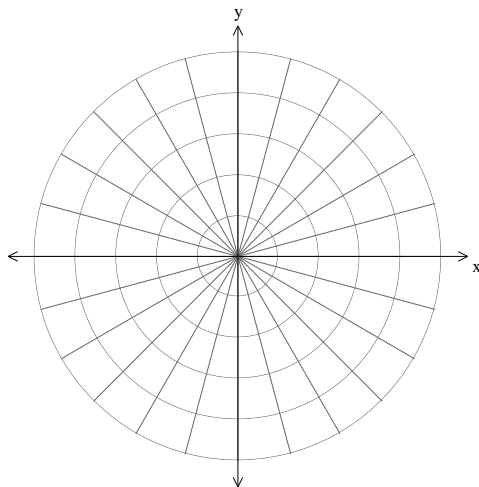
Examples:

- a) Convert $(3, 45^\circ)$ to rectangular coordinates. b) Convert $(-2, 2\sqrt{3})$ to polar coordinates.

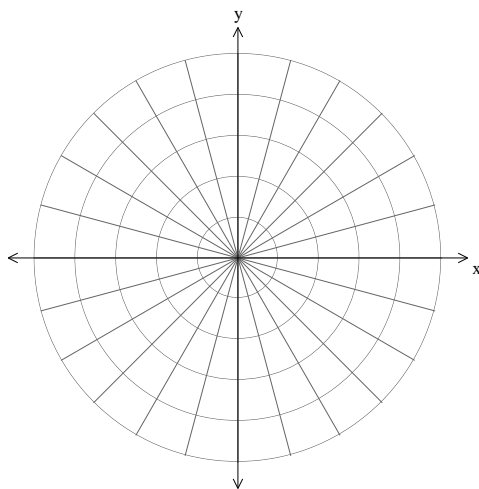
Graphing Polar Equations

Examples: Sketch the graphs of the following:

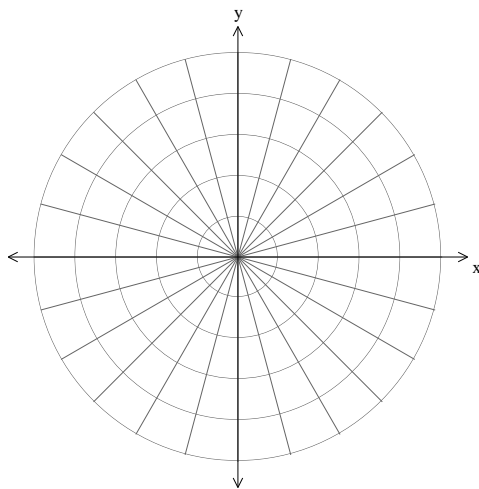
a) $r = 4 \sin \theta$



b) $r = \cos(2\theta)$



c) $r = 3 + 4 \sin \theta$



Examples: Convert equations from polar to rectangular form.

a) Convert $r = 3\sin\theta$ to a rectangular equation.

b) Convert $r = \frac{4}{1+\sin\theta}$ to a rectangular equation.

Examples: Convert equations from rectangular to polar form.

c) Convert $y = -2x + 5$ to a polar equation.

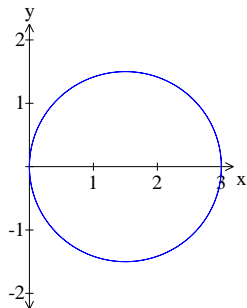
d) Convert $x^2 + (y-1)^2 = 1$ to a polar equation.

1) Lines through the origin are of the form $\theta = \alpha$.

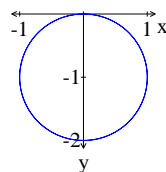
Vertical lines are of the form $r \cos \theta = a$.

Horizontal lines are of the form $r \sin \theta = a$.

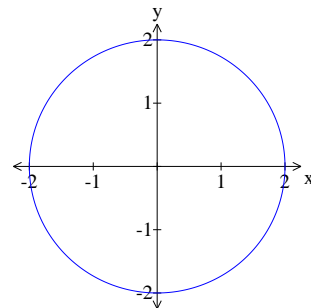
2) Circles come in three forms: $r = a \cos \theta$, $r = a \sin \theta$, and $r = a$.



$$r = 3 \cos \theta$$



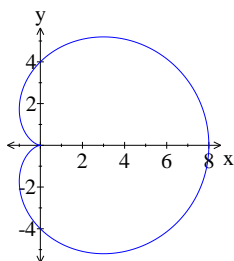
$$r = -2 \sin \theta$$



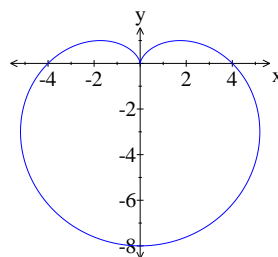
$$r = 2$$

3) Cardioids have the form $r = a \pm a \cos \theta$ or $r = a \pm a \sin \theta$.

Cardioids pass through the pole.



$$r = 4 + 4 \cos \theta$$



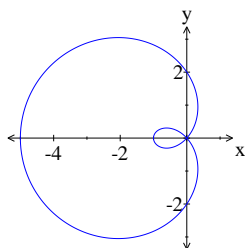
$$r = 4 - 4 \sin \theta$$

4) Limaçons have the form $r = a \pm b \cos \theta$ or $r = a \pm b \sin \theta$.

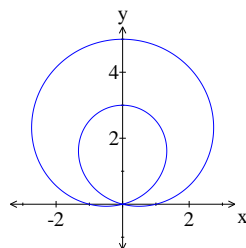
Limaçons have an inner loop if $0 < a < b$ and have no inner loop if $0 < b < a$.

The graph of a limaçon with an inner loop passes through the pole twice.

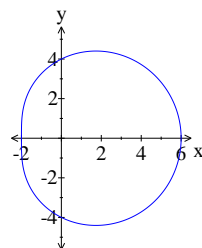
The graph of a limaçon with no inner loop does not pass through the pole.



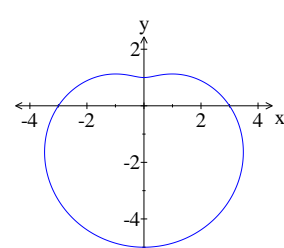
$$r = 2 - 3 \cos \theta$$



$$r = 1 + 4 \sin \theta$$

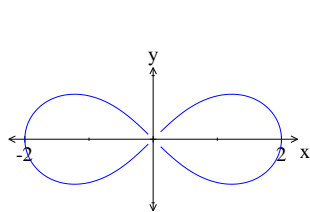


$$r = 4 + 2 \cos \theta$$

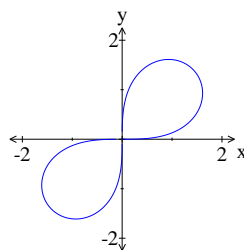


$$r = 3 - 2 \sin \theta$$

5) Lemniscates have the form $r^2 = a^2 \cos(2\theta)$ or $r^2 = a^2 \sin(2\theta)$.



$$r^2 = 4 \cos(2\theta)$$

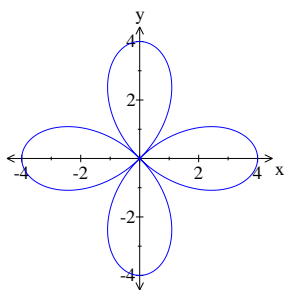


$$r^2 = 4 \sin(2\theta)$$

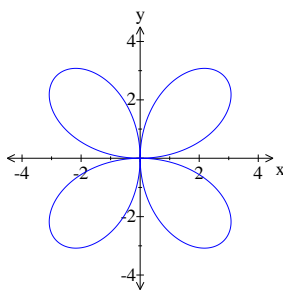
6) Roses have the form $r = a \cos(n\theta) + b$ and $r = a \sin(n\theta) + b$.

If n is even, there are $2n$ loops in the rose.

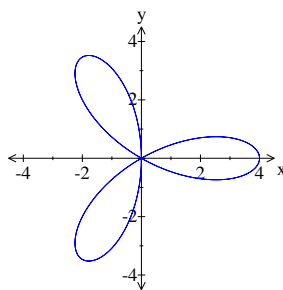
If n is odd, there are n loops in the rose.



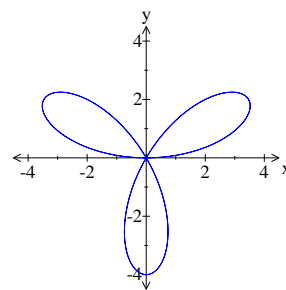
$$r = 4 \cos(2\theta)$$



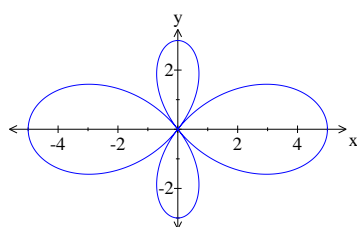
$$r = 4 \sin(2\theta)$$



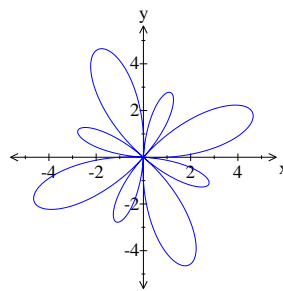
$$r = 4 \cos(3\theta)$$



$$r = 4 \sin(3\theta)$$



$$r = 4 \cos(2\theta) + 1$$



$$r = 4 \sin(4\theta) + 1$$